Trees and BSTs

BSTs: Extension 01

Paolo Camurati and Stefano Quer
Dipartimento di Automatica e Informatica
Politecnico di Torino

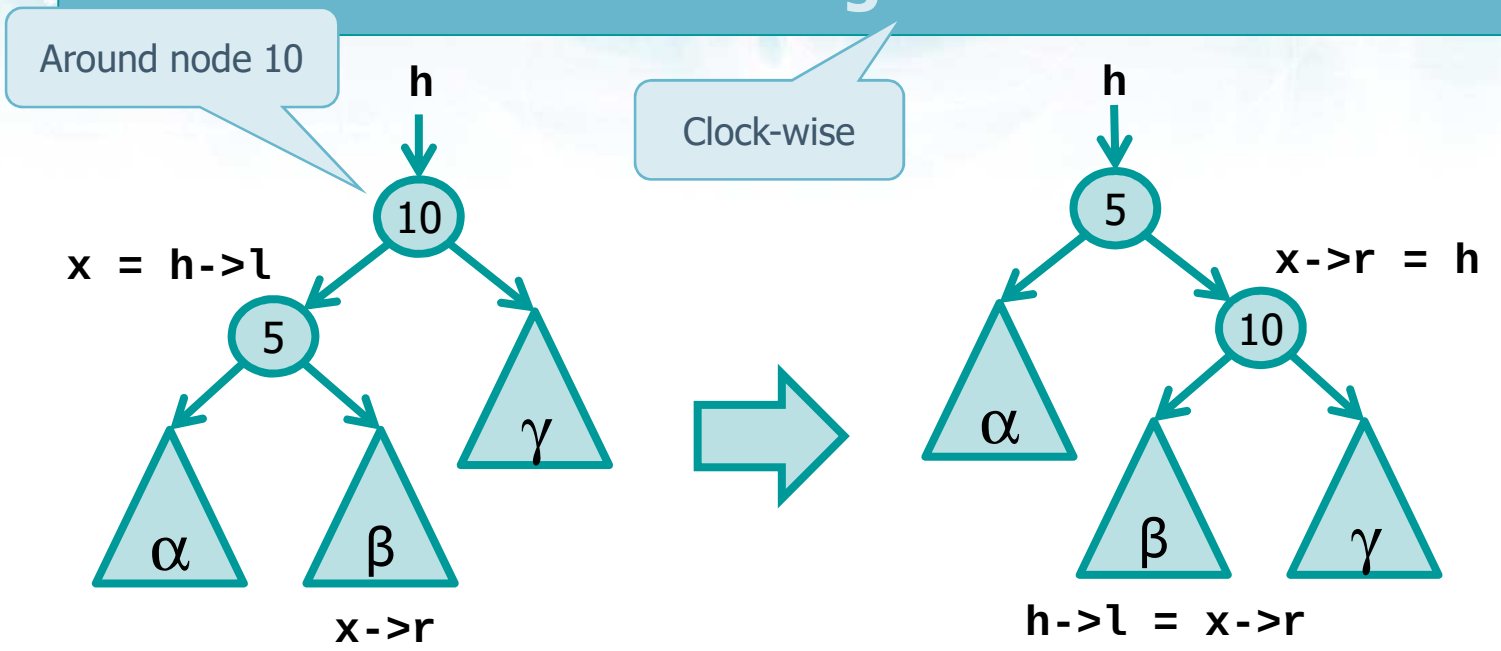
Root insertion

- Given a BST, nodes can be inserted
  - On the leaves, using the strategy previously analyzed
  - On the root, using a "root insertion" procedure
- Motivation
  - By inserting new nodes on the BST root, more recent key remains closer to the root
  - For the **locality** principle more recent keys have a higher probability to be addressed again soon
  - Thus, it is faster to search more recent keys speeding-up the BST manipulation

Root insertion

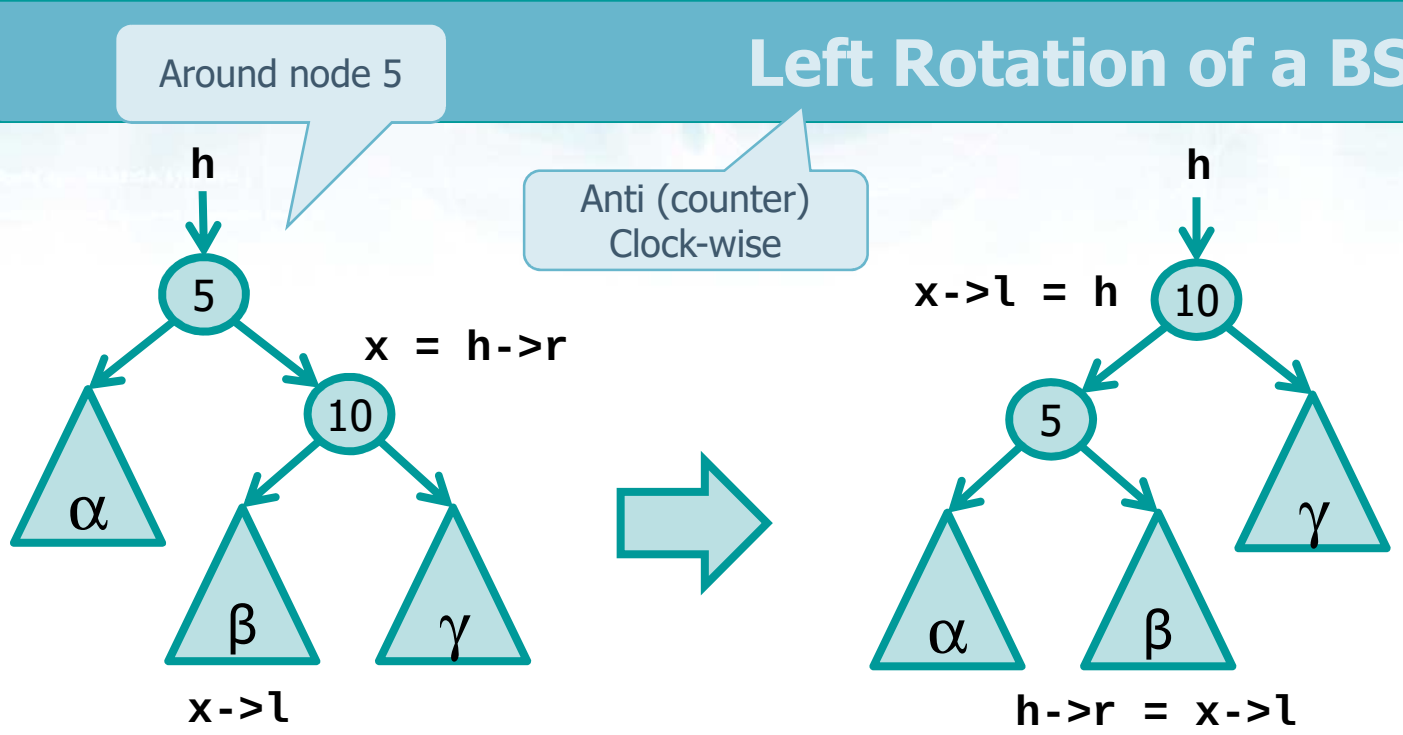
- Root insertions
  - Must maintain the BST property
  - Make use of local adjustments easy to implement and efficient to execute
- The core idea is to
  - Insert a new node onto a leaf
    - This can be done using the original standard insertion procedure
  - Move the node onto the tree root
    - This step maybe performed using **rotations**
    - There are two form of rotations: **Right** and **Left**

Right Rotation of a BST



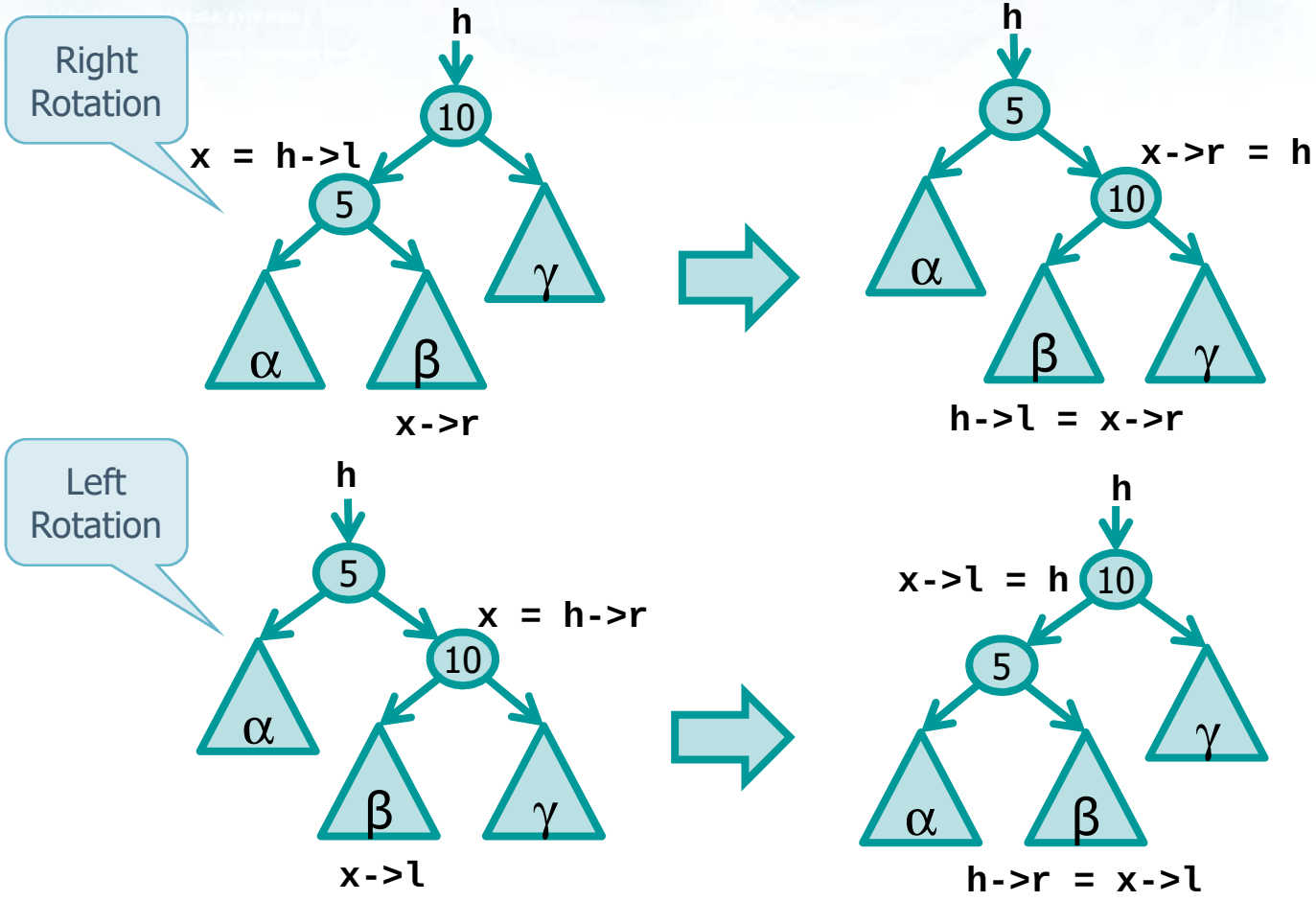

```
link rotR (link h) {
  link x = h->l;
  h->l = x->r;
  x->r = h;
  return x;
}
```

Left Rotation of a BST




```
link rotL(link h) {
  link x = h->r;
  h->r = x->l;
  x->l = h;
  return x;
}
```

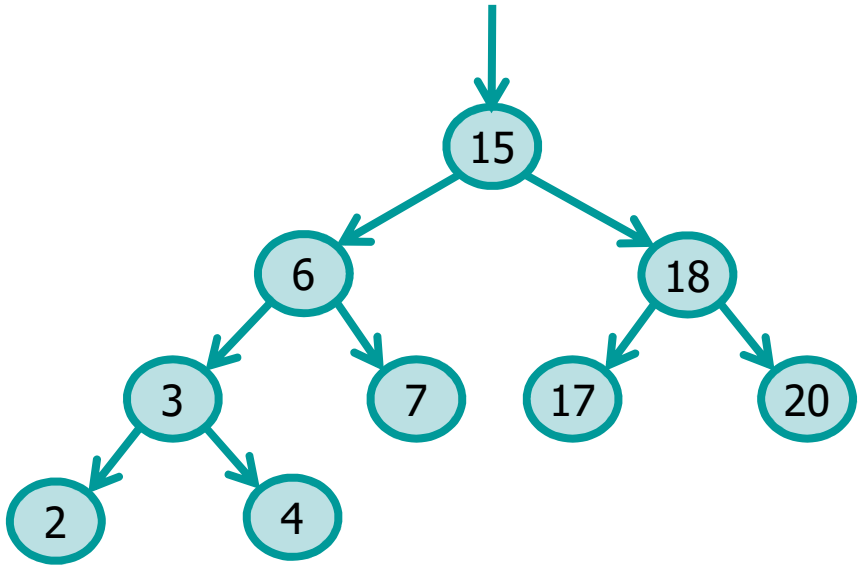
Right vs Left Rotation of a BST



Examples

- Given the following tree perform
  - A right rotation around node 6
  - A left rotation around node 18

Around node … its the node **on top**

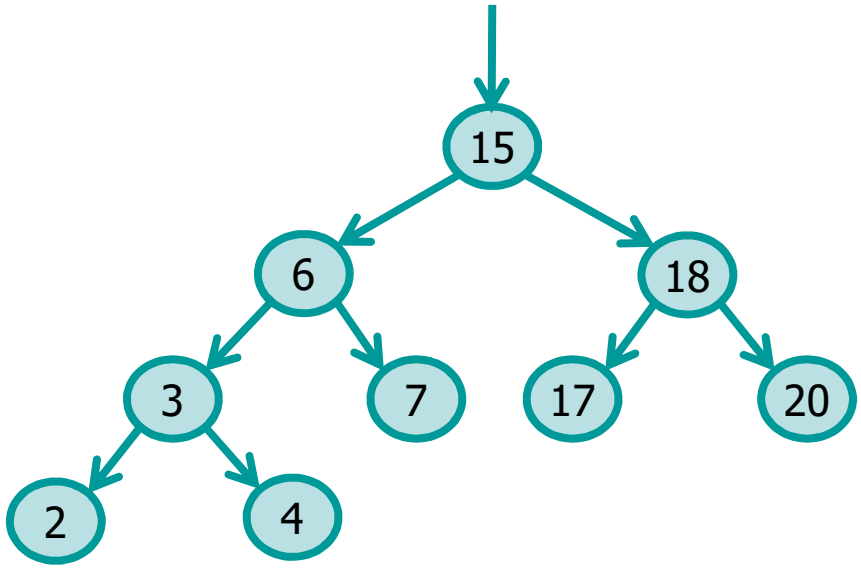


Root insertion

- To insert a new node on the root of a BST it is possible to use the following recursive procedure
  - Insert the new node into the appropriate sub-tree following the leaf insertion procedure
  - Rotate the node moving it upward, to force it onto the tree root

Examples

- Given the following tree perform the insertion **on the root** of the following keys
  - key = 16
  - Key = 5



Implementation

```
link insert_root_r (link root, Item x, link z) {
  if (root == z)
    return NEW (x, z, z);

  if (item_less(x, root->item)) {
    root->l = insert_root_r (root->l, x, z);
    root = rotR (root);
  } else  {
    root->r = insert_root_r (root->r, x, z);
    root = rotL (root);
  }

  return root;
}
```

Recur left
Rotate right

Recur right
Rotate left

Exercise

- Given an initially empty BST insert the following keys on the root
  - 20 12 9 30 36 31 33

Exercise

- Given an initially empty BST perform the following insertions (+) and extractions (–)
  - +10 +11 +21 +9 +18 +37 +25 + 7 +5 +39 +30 +6 +32 +17 +15 +13 +2 +24 –21 –18 –11 –10
  - Insert on leaves before (exercise 1)
  - Insert on root after (exercise 2)

Exercise

- Given an initially empty BST perform the following insertions (+) and extractions (–)
  - +A +S +E +R +C +H +I +N +G +X +M +P +L –A –R –I –G
  - Insert on leaves before (exercise 1)
  - Insert on root after (exercise 2)