

Introduction to Algorithms

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Algorithms

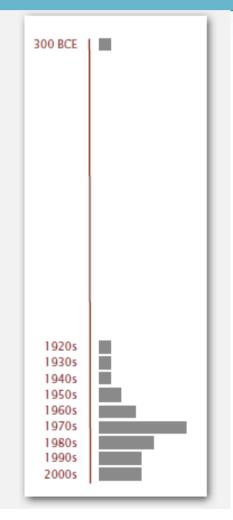
- Algorithms are defined as
 - Systematic and well-defined logical approach used to solve a problem
 - > Satisfy the following criteria
 - They receive input values
 - They produce output values
 - They are clear, unambiguous and executable
 - They terminate after a finite number of steps
 - They work on data structures

Programs

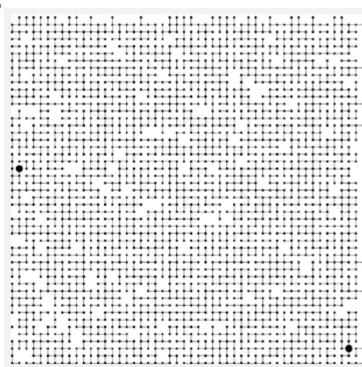
- Whenever a computer is used to execute an algorithm, the steps which lead to the solution should be properly encoded for the computer itself
- Programs and procedure
 - Definition of an algorithms using a defined programmig language
 - Finite sequence of instructions that solve a problem

"Programming is learned by writing programs"
Brian Kernighan

- Algorithms have ancient roots
 - > Euclid, Greek, IV cent, B.C.
 - > Al-Huarizmi, Persian IX cent., A.D.
 - Formalization by Church and Turing
 - Princeton University, 1930s
 - > Recent developments



- To do something otherwise impossible
 - > An example of netword connectivity
 - Are the two dark dots connected in this network?



Yes!

- Their impact is broad and far-reaching
 - ➤ Internet: Web search, packet routing, distributed file sharing
 - > Biology: human genome
 - > Computers: CAD tools, file systems, compilers
 - Graphics: virtual reality, videographics
 - Multimedia: MP3, JPG, DivX, HDTV
 - Social Networks: recommendations, news feed, advertisement
 - > Security: e-commerce, cell phones
 - Physics: particle collision simulation

- To unlock the secrets of life and of the universe by creating models
 - ➤ In many sciences computational models are replacing mathematical ones

```
f(x) = a_0 + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right)\sin \alpha \pm \sin \beta = 2 \sin \frac{1}{2} (\alpha \pm \beta) \cos \frac{1}{2} (\alpha \mp \beta)
```

20th century science Mathematical formulae (formula based)

```
int f (int **m, int n, int m) {
  int i, j, sum;
  sum = 0;
  for (i=0; i<n; i++) {
    for (j=0; j<m; j++) {
      sum+=m[i][j];
    }
  }
  return sum;
}</pre>
```

21th century science Computational model (algorithm based)

- For intellectual stimulation
 - "An algorithm must be seen to be believed."

Donald Knuth

- To become a proficient programmer
 - "I will, in fact, claim that the difference between a bad programmer and a good one is whether he cosiders his code or his data structures more important. Bad programmers worry about the code. Good programmers worry about data structures and their relatioships."

Linus Torvalds (creator of Linux)

For fun, for money, to increase speed, to process more data, ...

Types of problems

Decision problems

- > Problems with a yes/no answer
- > They may be
 - Decidable
 - There exists an algorithm that solves them
 - Undecidable
 - There is no algorithm that solves them

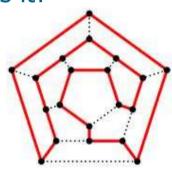
> Examples

- Given 2 integers x and y, does x exactly divide y?
- Given a positive integer x, is it prime?
- Given a positive integer n, do 2 positive and > 1 integers p and q exist such that n = pq?
- Determine whether a number is prime

Types of problems

Search problem

Does a valid solution exist and which one is it?



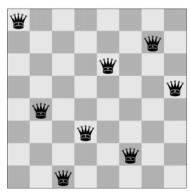


> Examples

- Hamiltonian cycle
 - Given an undirected graph, does a simple cycle spanning all vertices exist? Which one is it?
- Which one is the k-th prime number
- Given an array of integers, sort it in ascending order

Types of problems

- Verification problems
 - Figure 3 Given a solution (certificate), make sure that it is really one
 - > Examples
 - Sudoku
 - The eight queen problem



5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	m	4	8
1	9	8	ო	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

Copenhagen

🖒 Berlin

Stuttgart

Types of problems

- Optimization problems
 - > If a solution exists, which one is the best one?
 - > Examples

• Given a weighted directed graph, which is the shortest simple path, if it exists, between nodes i and j?