

01OGD Algorithms and Programming

22/02/2016 – Part I: Theory (12 points)

1. (1 point)

Given the following sequence of pairs, where the relation $i-j$ means that node i is adjacent to node j :

1-7, 2-1, 0-3, 4-0, 1-6, 3-4, 0-8, 1-3, 6-7, 10-3

apply an on-line connectivity algorithm with quick find, showing at each step the contents of the array and the forest of trees at the final step. Node names are integers in the range from 0 to 10.

2. (2.5 points)

Solve with unfolding the following recurrence equation:

$$\begin{aligned} T(n) &= T(n-1) + n^2 & n > 1 \\ T(1) &= 1 & n = 1 \end{aligned}$$

Remember: $\sum_{i=0}^{n-1} i^2 = \frac{n(n+1)(2n+1)}{6}$

3. (1 point)

Sort in ascending order with counting sort the following array of integers:

3 0 5 0 6 1 4 0 8 1 2 1 8 0 2

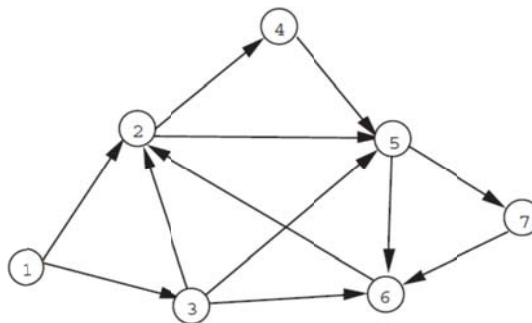
Show relevant intermediate steps.

4. (2 points)

Given the sequence of strings **In Bob Until Xeno Edge Graph Zeta Hotel You**, where each string's key is its first letter, identified by its index in the English alphabet (A=1, ..., Z=26), draw the final configuration of an initially empty hash table of size 19 where insertion of the previous sequence occurs. Assume open addressing with double hashing. Define suitable hash functions. Show relevant intermediate steps.

5. (1.5 + 1.5 points)

Given the following directed graph:

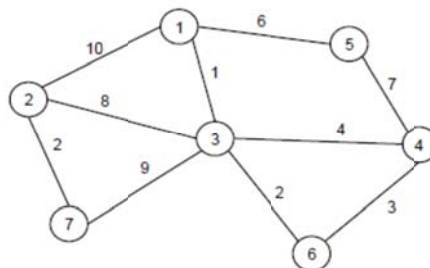


- visit it in breadth-first starting from node 1 (1.5 points)
- redraw it, labelling each edge as T (tree), B (back), F (forward), C (cross). Start from node 1 (1.5 points).

If necessary, consider nodes in alphabetical order..

6. (1.5 + 0.5 + 0.5 points)

Using Kruskal's algorithm starting from node 1, find a minimum-spanning tree for the following undirected and weighted graph. Show relevant intermediate steps (1.5 points).



Once a solution has been found, imagine to add a new edge (1-4) whose weight is 5. Is this new edge relevant to improve the solution's cost? Why (0.5 points)? Then imagine to add a new edge (1-7) whose weight is 5. Is this new edge relevant to improve the solution's cost? Why (0.5 points)?